1 Integer Program for Minimal Popular Vote

Let S denote the set of the District of Columbia together with all states (with the exception of Maine and Nebraska), let M_1 and M_2 denote the congressional districts of Maine, where $M := \{M_1, M_2\}$, and let N_1, N_2, N_3 denote the congressional districts of Nebraska, where $N := \{N_1, N_2, N_3\}$. In addition, let $D := M \cup N$ and let v_i denote the voter turnout for each $i \in S \cup D$. Furthermore, let $\bar{v}_M := \left\lfloor \frac{v_{M_1}+v_{M_2}}{2} \right\rfloor + 1$, $\bar{v}_N := \left\lfloor \frac{v_{N_1}+v_{N_2}+v_{N_3}}{2} \right\rfloor + 1$, $\bar{v}_i := \left\lfloor \frac{v_i}{2} \right\rfloor + 1$. Finally, let c_i denote the number of electoral votes for each $i \in S \cup D$. The optimal objective value of the following integer program gives the minimal possible popular vote for a given election year.

$$
\begin{aligned}
\text{minimize} \quad & \sum_{s \in S} \bar{v}_s x_s + \sum_{d \in D} z_d & \text{(1a)}\\
\text{subject to} \quad & \sum_{i \in S \cup D} c_i x_i + 2y_N + 2y_M \geq 270, & \text{(1b)}\\
& z_{M_1} - \left\lceil \frac{v_{M_1}}{2} \right\rceil x_{M_1} - \left\lfloor \frac{v_{M_1}}{2} \right\rfloor \leq 0, & \text{(1c)}\\
& z_{M_2} - \left\lceil \frac{v_{M_2}}{2} \right\rceil x_{M_2} - \left\lfloor \frac{v_{M_2}}{2} \right\rfloor \leq 0, & \text{(1d)}\\
& z_{N_1} - \left\lceil \frac{v_{N_1}}{2} \right\rceil x_{N_1} - \left\lfloor \frac{v_{N_1}}{2} \right\rfloor \leq 0, & \text{(1e)}\\
& z_{N_2} - \left\lceil \frac{v_{N_2}}{2} \right\rceil x_{N_2} - \left\lfloor \frac{v_{N_2}}{2} \right\rfloor \leq 0, & \text{(1f)}\\
& z_{N_3} - \left\lceil \frac{v_{N_3}}{2} \right\rceil x_{N_3} - \left\lfloor \frac{v_{N_3}}{2} \right\rfloor \leq 0, & \text{(1g)}\\
& \bar{v}_N y_N - z_{N_1} - z_{N_2} - z_{N_3} \leq 0, & \text{(1h)}\\
& \bar{v}_M y_M - z_{M_1} - z_{M_2} \leq 0, & \text{(1i)}\\
& \bar{v}_{M_1} x_{M_1} - z_{M_1} \leq 0, & \text{(1j)}\\
& \bar{v}_{M_2} x_{M_2} - z_{M_2} \leq 0, & \text{(1k)}\\
& \bar{v}_{N_1} x_{N_1} - z_{N_1} \leq 0, & \text{(1l)}\\
& \bar{v}_{N_2} x_{N_2} - z_{N_2} \leq 0, & \text{(1m)}\\
& \bar{v}_{N_3} x_{N_3} - z_{N_3} \leq 0, & \text{(1n)}\\
& z_i \leq v_i, \quad \forall i \in D, & \text{(1o)}\\
& z_i \geq 0, \quad \forall i \in D, & \text{(1p)}\\
& z_i \in \mathbb{Z}, \quad \forall i \in D, & \text{(1q)}\\
& x_i \in \{0,1\}, \quad \forall i \in S \cup D, & \text{(1r)}\\
& y_M \in \{0,1\}, & \text{(1s)}\\
& y_N \in \{0,1\}. & \text{(1t)}
\end{aligned}
$$

In a solution to the integer program above, for each $i \in S \cup D$ we have that $x_i = 1$ if our candidate wins, and $x_i = 0$ otherwise. For each $i \in D$, z_i is the number of votes won by our candidate in congressional district i. Finally, $y_M = 1$ if our candidate wins the state of Maine (and is therefore awarded an additional 2 electoral votes), and $y_M = 0$ otherwise. Similarly, $y_N = 1$ if our candidate wins the state of Nebraska (and is therefore awarded an additional 2 electoral votes), and $y_N = 0$ otherwise. Inequality (1b) ensures our candidate wins at least 270 required electoral votes. Inequalities (1c) – (1g) ensure that if a majority of voters in a given congressional district vote for our candidate then the corresponding electoral vote for the given congressional district is awarded to our candidate. Inequalities (1h) and (1i) ensure that if an additional 2 electoral votes are awarded to our candidate for the state of Maine (or Nebraska), then our candidate must have won at least the minimum majority vote in the corresponding state. Finally, inequalities (1j) – (1n) ensure that if an electoral vote is awarded to our candidate for a given congressional district in Maine or Nebraska, then our candidate must have won at least the minimum majority vote in the given district.